DYNAMICS OF NANOSECOND SPARK-GAP CHANNELS

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Abstract

S.I. Braginskii [1] showed that the resistive collapse of a very fast spark discharge in a gas is governed in large part by the radial expansion of a cylindrical shock wave, which rapidly increases the cross-sectional area of the conducting channel. Though this model has been shown to give good qualitative agreement with experiment by a number of authors, it contains assumptions that are not a priori justified. In particular, it assumes that electrical conductivity remains constant during channel expansion, implying that hydrodynamic, radiative, and thermal cooling are precisely offset by Joule heating. In this paper we show that data by Sorensen and Ristic [2] at the nanosecond timescale is not, in fact, well modeled by constant electrical conductivity. Instead, we find that conductivity must increase considerably during the first quarter cycle in order to agree well with their data. To better understand the energy balance of an expanding, Joule heated channel we have performed one-dimensional magnetohydrodynamic simulations to model in detail the transport of energy out of it. We find that at the nanosecond timescale, radiation and thermal transport are insufficient to keep the channel temperature from rising rapidly at early time.

I. THE BRAGINSKII MODEL

The phenomenology of spark-gap breakdown has been intensely studied for many decades. In particular, the early stages of this process, which determine the voltage threshold and time delay until the onset of breakdown are rather well understood over a wide range of parameters. Historically, there has been much less interest in the dynamics of the channel after it becomes sufficiently conducting to draw current, Joule heat, and become more conducting. Since the energy efficiency of some wide band microwave sources are driven by this process, however, we have attempted to study it An early model for the conduction phase was proposed by Rompe and Weitzel [3] in which they assumed that the energy resulting from Joule heating goes entirely into raising the temperature, and therefore the electrical conductivity of the channel. Channel expansion is neglected. Mesyats and Korshunov [4] showed that this model could give good agreement with experiment with the appropriate choice of initial conditions for the conducting channel. At the other extreme was the model first proposed by Drabkina [5] and refined by Braginskii in which the energy resulting from Joule heating goes primarily into channel expansion. This expansion, together with thermal conduction and/or radiation, keeps the temperature nearly constant. Electrical conductivity remains approximately constant in this model. Subsequent experimental work by a number of authors [2,6,7,8] suggests that the latter model as refined by Braginskii is most appropriate for modeling high voltage spark gaps.

The Braginskii model assumes that, as the initially very narrow channel Joule heats, its temperature and, therefore, its pressure, rapidly rises, causing it to expand rapidly, which, in turn, drives a strong, cylindrical shock wave into the undisturbed gas. It further assumes that hydrodynamic cooling associated with expansion, together with radiative cooling, are sufficient to keep the temperature of the conducting channel, and therefore its electrical conductivity,

$$a^2 = \left(\frac{4}{\pi\ \rho_0 \xi \sigma}\right)^{1/3} \int dt I^{2/3}\ , \qquad (1)$$

σ, approximately constant. These approximations result in an integral equation for channel radius,
where a is the radius of the conducting channel, ρ_0 is the density of the undisturbed gas, I is the current, and ξ is related to the γ of the gas,

$$\xi \cong K\left(1+\frac{1}{\gamma-1}\frac{1}{\dot{a}^2}\left(\ddot{a}a+\dot{a}^2\right)\right). \qquad (2)$$

Braginskii further assumes that the shock is strong so that K~0.9 and that both γ and the expansion velocity are approximately constant so that ξ is also approximately constant. For air and for nitrogen, he asserted that $\xi = 4.5$ corresponding to $\gamma \sim 1.25$. It is interesting to note that in order for the radial expansion to remain constant, the pressure must also remain constant. Furthermore, since we have also determined that the temperature must also remain approximately constant, we conclude that mass

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14. ABSTRACT
S.I. Braginskii [l] showed that the resistive collapse of a very fast spark discharge in a gas is governed in large part by the radial expansion of a cylindrical shock wave, which rapidly increases the cross-sectional area of the conducting channel. Though this model has been shown to give good qualitative agreement with experiment by a number of authors, it contains assumptions that are not a priori justified. In particular, it assumes that electrical conductivity remains constant during channel expansion, implying that hydrodynamic, radiative, and thermal cooling are precisely offset by Joule heating. In this paper we show that data by Sorensen and Ristic [2] at the nanosecond timescale is not, in fact, well modeled by constant electrical conductivity. Instead, we find that conductivity must increase considerably during the first quarter cycle in order to agree well with their data. To better understand the energy balance of an expanding, Joule heated channel we have performed one-dimensional magnetohydrodynamic simulations to model in detail the transport of energy out of it. We find that at the nanosecond timescale, radiation and thermal transport are insufficient to keep the channel temperature from rising rapidly at early time.

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from the shocked region must be constantly ejected into the conducting channel in order to keep its density approximately constant. With these approximations for ξ it becomes possible to determine channel radius as a function of time from the current as a function of time, providing one also knows the initial radius of the channel, together with its electrical conductivity.

II. THE EXPERIMENT OF SORENSEN AND RISTIC

In 1977, a simple and elegant experiment was performed by Sorensen and Ristic that offered a straightforward comparison of theory and experiment. They terminated a cylindrical transmission line with a very low inductance, simple geometry, self-breaking switch and obtained a time-resolved measurement of the reflected voltage as the switch resistance collapsed. They then used the Braginskii model to relate the measured voltage to switch resistance and channel radius. Their simple circuit is shown schematically in Figure 1.

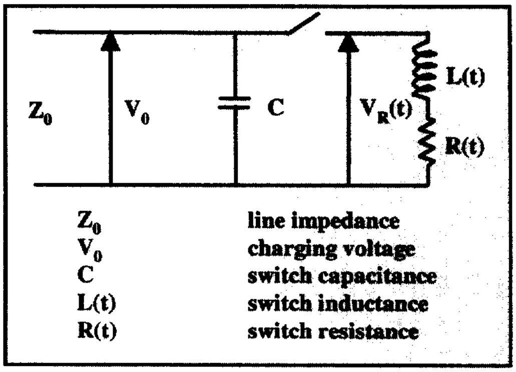

Figure 1. Schematic of Sorensen and Ristic circuit.

Under the assumptions that the transmission line transit time is long compared to the opening time of the switch and that the charge time is slow compared to the discharge time, the circuit obeys the following equation:

$$\frac{R_0(t)}{Z_0} = -(V_0 + V_R(t)) \times \times\left(V_R(t) + CZ_0\left(\frac{dV_R(t)}{dt}\right)\right)^{-1} \tag{3}$$

where $V_R(t)$ is the reflected voltage. This reflected voltage was measured near the switch from which it was possible to determine the switch resistance as a function of time. They obtained data for nitrogen at a number of different voltages, gap spacings, and line impedances, from which they were able to construct a simple power law fit for switch resistance as a function of time,

$$\frac{R(t)}{Z_0} = 2x10^4\left(\frac{p^{1/2}}{EZ_0^{1/3}}\right)^3, \tag{4}$$

where p is the nitrogen pressure in atmospheres and E is the electric field in units of 10^4 kV/cm.

III. AN ANALYTIC MODEL

The above procedure can easily be inverted in order to obtain switch current and reflected voltage from switch resistance as a function of time. Since the Braginskii model gives the latter for known switch current, it is possible to use the data of Sorensen and Ristic to test the model.

The equation for reflected voltage in the line, following Sorensen and Ristic, but not neglecting inductance inductance a priori, contains the unknown current, resistance and inductance, together with their time derivatives,

$$\frac{dV_R(t)}{dt} = -\frac{1}{CZ_0}\left[\left(1 + \frac{Z_0}{R(t)}\right)V_R(t) + + \frac{Z_0}{R(t)}\left(V_0 - L(t)\frac{dI(t)}{dt} - I(t)\frac{dL(t)}{dt}\right)\right] \tag{5}$$

An equation for current is obtained by observing that the charging voltage minus the magnitude of the reflected voltage must equal the sum of the voltage drops across the resistance and the inductance,

$$\frac{dI(t)}{dt} = \frac{V_R(t) + V_Z - R(t)I(t) - \frac{dL(t)}{dt}I(t)}{L(t)}. \tag{6}$$

Inductance and its time derivative are related to channel radius, a(t), and its time derivative

$$L(t) = 2x10^7\,\ell \ln\left(\frac{a_0}{a(t)}\right),$$

$$\frac{dL(t)}{dt} = \frac{-2x10^{-7}\,\ell}{a(t)}\frac{da(t)}{dt}. \tag{7}$$

where •is the length of the switch. Closure of this system of equations is obtained is obtained by differentiating Eq. 1 of Braginskii, where σ is assumed constant.

$$\frac{da(t)}{dt} = \frac{const}{a(t)}I^{2/3}, \quad R(t) = \frac{\ell}{\sigma\pi a^2(t)}. \tag{8}$$

A Runge-Kutta solution for equations 5-8 is shown in Figure 2 for E = 400 kV/cm, Z_0 = 29 W, p = 5.9 atm, ρ_0 = 8.73 kg/m3, and a_0 = 1 micron. Also, we have taken σ = $2x10^4$ Mho/m corresponding to the value proposed by Braginskii and by Martin [8]. This curve is compared to results obtained from Eq. 4 with the same parameters. It is interesting to note that these curves have very different asymptotes and that they have very different slopes during the first few hundred picoseconds. Though it is interesting

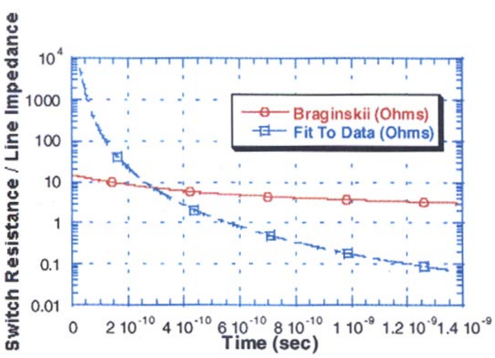

Figure 2. Compares the results of the Braginskii model of the Sorensen and Ristic experiment to data.

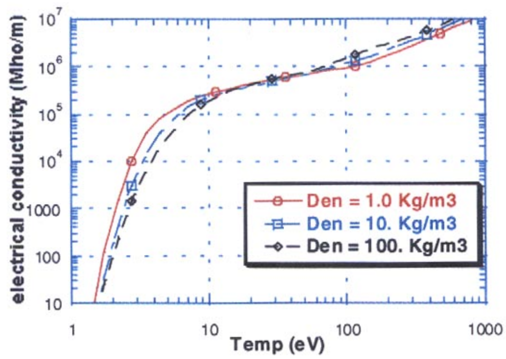

Figure 4. This shows electrical conductivity of nitrogen as a function of temperature for three different densities.

to note that Cary and Mazzie [9] inferred a less steep resistance collapse than Sorensen and Ristic in a similar experiment, we assert that these curves are sufficiently different to imply a problem with the Braginskii model.

In a paper that considered a very different set of switch parameters, Brambilla [10] argued that switch performance was strongly influenced by the initial radius of the conducting channel. In Figure 3 we show

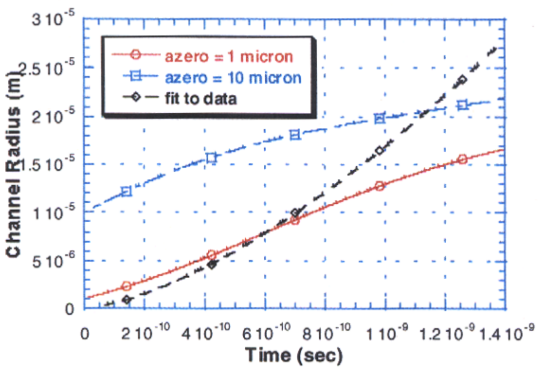

Figure 3. Radius as a function of time with an initial radius of 1 and 10 microns. This is compared to the Sorensen and Ristic fit using a conductivity of 2.2×10^4.

radius as a function of time obtained in the manner of Fig. 2 and using the same parameters except for the initial radius which is allowed to be both 1 micron and 10 microns. From this we note that neither curve matches the Sorensen and Ristic fit to data very well, though the curve with 1 micron initial radius is much closer at early time.

One possible explanation for the divergence of the Braginskii model from the data at late time is related to the fact that the model assumes constant electrical conductivity. In particular, it assumes a onductivity of approximately 2.22×10^4 Mho/m, which corresponds to a temperature of approximately 5 eV. Figure 4 shows electrical conductivity as a function of both temperature and density from the Los Alamos Sesame tables. The first things we note are that the dependence on density is modest within the density range of interest and that conductivity assumed by Braginskii corresponds to a temperature of approximately 5 eV. It is equally interesting to note that, at this temperature, conductivity is varying very rapidly. This implies that, in order for the electrical conductivity to remain approximately constant, the temperature must remain precisely constant. There is no reason to believe a priori that this is the case. In fact, one suspects from Fig. 2 that an electrical conductivity rising with time would match the experimental data considerably better.

IV. NUMERICAL SIMULATIONS

We conclude from the above that the model of Braginskii isn't able to adequately reproduce the data from the simple experiment of Sorensen and Ristic, and we speculate that this discrepancy might be removed if the model allowed the temperature to increase with time. To better understand the detailed dynamics and energy balance we have performed detailed one-dimensional (1-D) MHD simulations with MACH2 [11]. These initial simulations covered only the first 600 psec due to difficulties with magnetic field transport through the undisturbed layer of gas, but they were sufficient to suggest problems with earlier models. Figure 5 shows

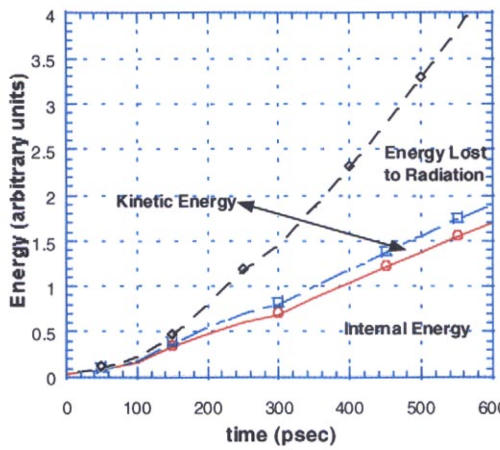

Figure 5. Energy balance from a 1-D MACH2 simulation.

the energy balance from such a (1-D) simulation, in which the approximate current vs time profile from the analytic model is imposed as a boundary condition upon a channel with the same initial conditions. It is interesting to note that during the first 150 psec little energy is lost to radiation, whereas after that time approximately half the energy deposited by Joule heating in the channel is radiated away. This corresponds to a channel temperature that rose from an initial 2.5 eV (chosen so that it would be conducting) to around 15 eV, after which it remained approximately constant. This suggests that Braginskii's hypothesis that radiation losses would keep the temperature and, therefore, conductivity, constant is correct. However, Braginskii and others assumed a value of conductivity that is approximately an order of magnitude lower than the simulations imply. These differences would lead to a channel conductivity that would increase considerably faster than predicted by the model, consistent with Sorensen and Ristic data.

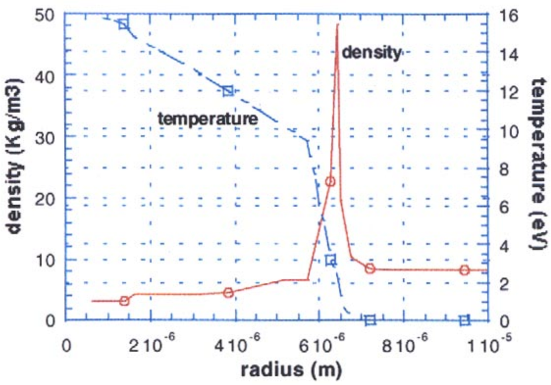

Figure 6. Snapshot of radial density and temperature profiles at t = 600 psec.

Figure 6 shows the density and temperature profiles at t = 600 psec from the same simulation as Fig. 5. The fact that the density within the channel is approximately half the undisturbed gas density despite the fact that its radius has increased by a factor of 6 suggests that Braginskii's implied hypothesis that density should remain approximately constant is correct. However, by 600 psec there is considerable structure to the temperature profile, suggesting that at late time the simple model, which is based on the hypothesis that the regions can each be categorized by a single temperature and density, is beginning to break down.

V. CONCLUSIONS

We find that the Braginskii model does a good job of qualitatively describing the features of the Sorensen and Ristic experiment. However, the experimental conductivity increases somewhat more rapidly than the model, a discrepancy that could be understood if channel temperature were to increase with time, rather than remaining constant, as Braginskii hypothesizes. 1-D simulations of channel evolutions offer insight into this discrepancy. In particular, we find that rapid temperature rise to a value somewhat higher than Braginskii predicted explains the more rapid rise during the first 150 psec. At later time we find significant density and temperature structure implying that the simple model no longer applies.

The next steps in the simulations are to self-consistently couple Eqs 5-8 to the code's external circuit solver and to run them to late time to determine integrated losses within the switch.

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